**Functions and relations**

**Slide 1**

A relation is a connection between elements of an input set and an output set.

x

y

z

m

p

q

r

p

q

r

p

q

x

y

z

m

n

(b)

(c)

(a)

The input set is the Domain. The output set is the Codomain.

Range is a set of elements in the codomain that has an element mapped to them in the domain.

**Slide 2**

A function is a relation where each input has only one output.

x

y

z

m

p

q

r

p

q

r

p

q

x

y

z

m

n

(b)

(c)

(a)

a, b and c are all relations

only a and b are functions

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Types of functions

One-to-One (Injective): each element in the domain map to a unique image in the codomain.

No two elements in the domain have the same image in the codomain.

p

q

r

y

a

b

c

**Slide 4**

Onto (Subjective): every image in the codomain is mapped from at least one element in the domain.

p

q

a

b

c

The codomain is the same as the Range.

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Bijective: A function that is both injective and subjective.

Each element in the domain has only its unique image in the codomain

Each element in the codomain has a preimage in the domain

p

q

y

a

b

c

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Example

Given f(x) = x² - 3x + 1

1. Find f(0), f(1) and f(3)
2. Find f(x+1)
3. is f(x) and injective function?

Solution

f(x) = x² - 3x + 1

1. f(0) = (0)² - 3(0) + 1 = 1

          f(1) = (1)² - 3(1) + 1 = -1

          f(3) = (3)² - 3(3) + 1 = 1

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1. f(x) = x² - 3x + 1

f(x+1) = (x + 1)² - 3(x + 1) + 1

= x² + 2x + 1 - 3x - 3 + 1

f(x+1) = x² - x - 1

1. f(0) = f(3) = 1,

f(x) is NOT injective

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Composite function

A function that takes element from a set and maps it to an image in another set passing through and intermediate function

Composite functions are formed from combination of two or more functions

g: A → B

f: B → C

f o g : A → C

f o g = f(g(x))

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Example

If f(x) = 3x - 2, g(x) = x² + 1 and h(x) =

Find:

1. f(g(2))
2. h o f (2)
3. h o g o f (-1)
4. f o g

Solution

1. f(g(2))

            g(2) = 2² + 1 = 5

            f(g(2))= f(5) = 3(5) - 2 = 13

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1. h o f (2)

h o f (2) = h(f(2))

f(2) = 3(2) - 2 = 6 - 2 = 4

h(f(2)) =h(4)

h(4) = = ⅘

 h o f (2) = 4/5

1. h o g o f (-1) = h(g(f(-1)))

f(-1) = 3(-1) - 2 = -3 - 2 = -5

g(f(-1))= g(-5) = (-5)² + 1 = 26

h(g(f(-1))) = h(26) = = 26/27

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1. f o g = f(g(x))

f(x² + 1) = 3(x² + 1) - 2 = 3x² + 1

f o g = 3x² + 1

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Inverse Function

A function that takes an image in the codomain and returns the preimage in the domain.

It reverses the operation performed on the image to find the preimage.

p

q

a

b

c

**Slide 13**

Steps for finding inverse function

* Write y = f(x)
* Make x the subject of the equation
* Replace y with x and x with inverse y-1

Example

Given

f(x) = 2x  -  3

1. Find the domain of f-1(x)
2. Find f-1(4)

**Slide 14**

Solution

 Write y = f(x)

y =  2x  -  3

Make x the subject of the equation

y(x-3) = 2

yx - 3y = 2

x =

Replace y with x and x with inverse y-1

y-1 =

f-1(x) =

The domain of f-1(x) is the values of x for which f-1(x) exists.

Domain of f-1(x) => (x < 0) U (x > 0)

Domain of f-1(x) = (-ꝏ, 0) U (0, ꝏ)

* 1. f-1(4).

f-1(4) = =

f-1(4) = 3

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Practice Questions

The functions f:x → x2 + 1 and g:x →5−3x are defined on the set of the real numbers, R.

1. Find f(-2)
2. g o f (1)
3. State the domain of f-1(x)
4. the inverse of f ;
5. Find g-1(1) and g-1(2)